Cambridge International Examinations
Cambridge Pre-U Certificate

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2016**

MARK SCHEME

Maximum Mark: 80

Published

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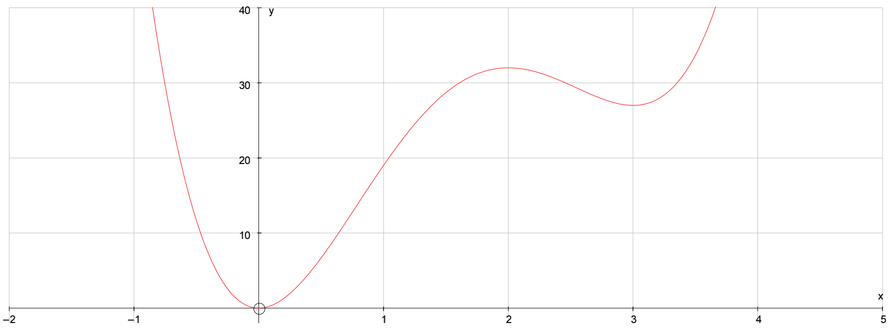
Question		Answer	Marks
1		State $m = -\dfrac{1}{5}$	B1
		Form equation $(y - 11) = (\text{their } m)(x - 1)$ or $11 = (\text{their } m)(1) + c$	M1
		Obtain $y = -\dfrac{1}{5}x + \dfrac{56}{5}$ or equiv decimal form as final answer	A1 [3]
2	(i)	Obtain $4\sqrt{20}$ or $4\sqrt{2}\ \sqrt{5}\times\sqrt{2}$	B1
		Obtain $8\sqrt{5}$	B1 [2]
	(ii)	Obtain $10\sqrt{5}$ or $5\sqrt{5}$	B1
		Obtain $15\ \sqrt{5}$	B1 [2]
3		Solve equation to obtain critical points	M1
		Obtain -5 and $\dfrac{4}{3}$	A1
		Show or imply method to obtain inequality, e.g. graph, table of signs	M1
		State $x < -5$ **or** $x > \dfrac{4}{3}$ (ft critical points).	A1ft [4]
4	(i)	Obtain 8, 11, 14	B1 [1]
	(ii)	Use correct formula $a + (n - 1)d = 254$	M1
		Obtain 83	A1 [2]
	(iii)	Use correct sum formula for AP	M1
		Obtain $\dfrac{500}{2}(2(8)+(500-1)3)$	A1
		Obtain 378 250 cao	A1 [3]
		<u>Alternative method:</u>	
		Obtain $8 + 499(3) = 1505$ and use correct $\dfrac{n}{2}(a+l)$	M1
		Obtain $\dfrac{500}{2}(8+1505)$	A1
		Obtain 378 250 cao	A1
5		State (3, 0)	B1
		Obtain or imply equation of the form $k \pm 9 = \pm 25$	M1
		Obtain $k + 9 = 25$	A1
		Obtain $k = 16$	A1 [4]

Question		Answer	Marks
6	(i)	Attempt to differentiate by reducing powers by one	M1
		Obtain $12x^3 - 60x^2 + 72x = 0$	A1
		Factorise x and attempt to solve a 3 term quadratic (but condone cancellation of x)	M1
		Obtain (0, 0), (2, 32), (3, 27)	A1
		Obtain the second derivative or compare gradients or y values either side of each point.	M1
		$36x^2 - 120x + 72$ must be used with either substitution of the relevant x values, or the final values 72, − 24 and 36 must be shown and similarly for comparison of gradients.	
		Conclude (0, 0) min, (2, 32) max, (3, 27) min (condone incorrect or no y values for this mark).	A1 [6]
	(ii)	Generally correct shape of a quartic, two min and one max. Stationary points marked OR correct $y = 27$ and $y = 32$ shown clearly	M1 A1
			
		$27 < k < 32$	A1 [3]
7	(i)	Range of f : $f(x) \geq 2$ Range of g is all real numbers	B1 B1 [2]
	(ii)	Obtain $(4x + 3)^2 + 2$ and $4(x^2 + 2) + 3$ Obtain $16x^2 + 24x + 11 = 4x^2 + 11$ Attempt to solve quadratic to obtain a value for x Obtain $x = 0$ and $x = -2$	M1 A1 M1 A1 [4]
	(iii)	Possibilities are $x \geq 0$ or $x \leq 0$. Either $y = \sqrt{x-2}$ or $y = -\sqrt{x-2}$ as appropriate for the domain	B1 B1* [2]
8	(a)	Use integration by parts with $f(x) = x$ and $g'(x) = e^{-x}$ Obtain $-xe^{-x} - e^{-x}$ Substitute limits in the correct order with subtraction. This must be seen if wrong answer obtained. Obtain $1 - \dfrac{2}{e}$ with no sight of decimals.	M1 A1 M1 A1 [4]

Question	Answer	Marks
(b)	Use $u = x + 1$ and substitute into the given integral	M1
	Obtain $\int \frac{u-2}{u}\,du$	A1
	Simplify to two terms and integrate or use by parts if integrating u^{-1} and differentiating $(u - 2)$	M1
	Obtain $x + 1 - 2\ln\|x + 1\| + C$ (A0 for omission of mod signs or + C)	A1 [4]
	Alternative method 1:	
	Obtain $1 + \frac{k}{x+1}$	M1
	Obtain $1 - \frac{2}{x+1}$	A1
	Attempt to integrate to obtain $x + k\ln(x + 1)$	M1
	Obtain $x - 2\ln\|x + 1\| + C$ (A0 for omission of mod signs or + C)	A1
	Alternative method 2:	
	Use parts on $(x - 1)(x + 1)^{-1}$ and obtain $(x - 1)\ln(x + 1)$ with a valid attempt at $\int \ln(x+1)\,dx$	M1
	Find $\int \ln(x+1)\,dx$, dealing with $\int \frac{x}{x+1}\,dx$	M1
	Obtain $(x - 1)\ln(x + 1) - (x + 1)\ln(x + 1) + (x + 1)$	A1
	Obtain $x - 2\ln\|x + 1\| + C$	A1
9	Set up at least 2 equations : $4 + 2\mu = 35 - 5\lambda$, $7 + 3\mu = 6 + 2\lambda$, $3 + 7\mu = 14 + 3\lambda$,	M1
	Find a value for λ or μ from two of them	M1
	Obtain $\mu = 3$, $\lambda = 5$ from the first two ($\mu = 5$, $\lambda = 8$ from last two; $\mu = 3.61$, $\lambda = 4.76$ from the first and last)	A1
	Demonstrate inconsistency in third eqn, e.g. $7 \times 3 - 3 \times 5 = 6 \neq 11$ **and** state do not intersect. This requires correct values for λ and μ $(3 + 7(3) = 24 \neq 14 + 3(5) = 29$ or $14 \neq -5)$	M1*
	Show the direction vectors are not multiples of each other **and** state they are not parallel	B1*
	OR find angle between direction vectors (= 69.498°) **and** state not parallel **OR** find dot product (= 17) **and** state is not equal to 1 and therefore not parallel)	
	State skew (requires accurate previous working)	depB1 [6]

Question	Answer	Marks
10 (i)	Attempt use of product rule to produce an expression of the form $k\ln(2y+3) \quad + \quad \dfrac{\text{linear in } y}{\text{linear in } y}$	M1
	Obtain $\ln(2y+3)$	A1
	Obtain $\ldots + \dfrac{2(y-4)}{2y+3}$ or unsimplified equiv	A1 [3]
	<u>Alternative method:</u>	
	Attempt use of product rule to produce $1 = \dfrac{dy}{dx}(\ln(2y+3)) + \dfrac{(y-4)\dfrac{2dy}{dx}}{2y+3}$	M1
	Obtain $\dfrac{dy}{dx} = \dfrac{2y+3}{2y-8+(2y+3)\ln(2y+3)}$	A1
	Obtain $\dfrac{dx}{dy} = \dfrac{2y-8+(2y+3)\ln(2y+3)}{2y+3}$	A1
(ii)	Attempt to find value of y for which $x = 0$	M1
	Obtain $y = -1$ and $y = 4$	A1
	Substitute $y = -1$ into attempt from part (i) or into their attempt (however poor) at its reciprocal SR. -10 without working M1A0. Other incorrect answers with no working M0	M1
	Obtain – 0.1 (dependent on correct answer from (i))	depA1
	Substitute $y = 4$ into attempt from part (i) or into their attempt (however poor) at its reciprocal. SR. ln 11 without working M1A0. Other incorrect answers with no working M0	M1
	Obtain $\dfrac{1}{\ln 11}$ (dependent on correct answer from (i))	depA1 [6]

Question	Answer	Marks
11 (i)	Use $\sin\left(\theta+\frac{\pi}{3}\right)=\sin\theta\cos\frac{\pi}{3}+\cos\theta\sin\frac{\pi}{3}$	B1
	(Award even if in incorrect expansion of $\sin^2\left(\theta+\frac{\pi}{3}\right)$)	
	Expand $\sin^2\left(\theta+\frac{\pi}{3}\right)$ to obtain a term involving $\sin\theta\cos\theta$	M1
	Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1
	Obtain $\frac{\sqrt{3}}{4}\sin 2\theta$ **AG**	A1
		[4]
	<u>Alternative method</u>	
	Use $\sin^2\theta=\frac{1}{2}(1-\cos 2\theta)$	B1
	Use $\cos(2\theta+\frac{2}{3}\pi)=\cos 2\theta\cos\frac{2}{3}\pi-\sin 2\theta\sin\frac{2}{3}\pi$	B1
	Substitute and evaluate expression	M1
	Obtain $\frac{\sqrt{3}}{4}\sin 2\theta$ **AG**	A1
(ii)	Use the result in (i) to obtain an equation in $\sin 2\theta$	M1
	Obtain $\sin 2\theta=\frac{-1}{\sqrt{3}}$	A1
	Use correct order of operations to obtain θ from an eqn in $\sin 2\theta$	M1
	Obtain any two correct angles	A1
	Obtain answers rounding to – 0.308, 2.83, −1.26 1.88	A1
		[5]

Question	Answer	Marks
12	State $\frac{\mathrm{d}x}{\mathrm{d}t}$	B1
	State $-\frac{k}{\sqrt{x}}$ (award B1 for $\frac{k}{\sqrt{x}}$ if $k = -0.1$)	B1
	Separate variables and integrate both sides, raising the powers by 1	M1
	Obtain $\frac{2}{3}x^{\frac{3}{2}} = -kt + C$	A1
	Substitute $x = 4$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = \pm\, 0.05$ to find k.	M1*
	Obtain $k = 0.1$	A1
	Substitute $t = 3$ and $x = 4$ to find C	depM1
	(dependent on a value for k obtained from using $x = 4$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = \pm\, 0.05$)	
	Obtain C = 5. 63(3333...) or $\frac{169}{30}$	A1
	or $\frac{169}{3}$ from $\frac{20}{3}x^{\frac{3}{2}} = -t + C$ or $-\frac{169}{30}$ if + c is placed on LHS	
	Substitute $x = 0.01$ into their solution provided of form $p\,x^{\frac{3}{2}} = \pm\,\mathrm{m}t + \mathrm{C}$ to find t	M1
	Obtain $t = 56.3$ or 56 days	A1
		[10]
	SR if $\frac{\mathrm{d}x}{\mathrm{d}t} = k\sqrt{x}$ award a maximum of B1 M3	
	SR if $-\frac{k}{\sqrt{x}}$ stated then $k = -0.1$ leads to final correct answer deduct A1 for k and A1 for the final answer = 8/10	